



ESTIMATION OF FIXED EFFECT MODELS ON TREE GROWTH VARIABLE PREDICTION IN A MIXED FOREST STAND, EDO STATE, NIGERIA

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ABSTRACT

This study evaluated the performance of six fixed-effect growth models, Logistic, Chapman-Richards, Mitscherlich, Gompertz, Polynomial, and Hossfeld IV for predicting tree growth dynamics in a mixed forest stand in Edo State, Nigeria. Accurate growth modeling is essential for sustainable forest management, particularly in heterogeneous stands where species interactions and site variability influence growth trajectories. Data were collected from established permanent sample plots representing the mixed forest conditions of the study area. Each model was fitted using fixed-effect estimation techniques, and their predictive accuracy was assessed using the coefficient of determination (R^2), root mean square error (RMSE), Akaike Information Criterion (AIC), and Bayesian Information Criterion (BIC). Results revealed notable differences in the models' ability to describe the growth patterns of trees within the mixed stand. Among the six candidate models, the Chapman-Richards function consistently outperformed the others, exhibiting the highest R^2 (76.19%) and the lowest RMSE (4.498), AIC (330.78), and BIC (334.28) values respectively, indicating superior goodness-of-fit and greater predictive reliability. The Logistic and Gompertz models provided moderately accurate estimates but fell short compared to the Chapman-Richards model, while the Polynomial, Mitscherlich, and Hossfeld IV (worst) models showed comparatively weaker performance and higher information criteria values. The findings underscore the suitability of the Chapman-Richards model for describing tree growth in mixed forest ecosystems in Edo State, emphasizing its ability to capture asymptotic growth behavior and structural complexity. This study provides a robust empirical basis for selecting appropriate growth models in tropical mixed forests and contributes to improved forest planning, yield prediction, and management decision-making in Nigeria. Future studies should incorporate longer-term monitoring data and additional ecological variables to enhance model robustness and predictive accuracy.

Keywords: Chapman-Richard, forest, fixed effect, *Terminalia superba*, model

INTRODUCTION

Fixed effect models are used to estimate the influence of factors like tree species, age, and environmental variables (e.g., soil and climate) that are constant, fixed effects are static parameters, that is to say they do not vary (Aishan, 2016; Clarke, 2025), these models remain valuable in many forestry applications especially when the variability between groups is minimal or when the dataset is balanced and well-structured. For example, (Kachamba and Eid (2016) demonstrated the effectiveness of

fixed effect models for estimating merchantable stem volume in hardwood stands. His study showed that with appropriate transformations and inclusion of variables like diameter at breast height (DBH) and total tree height, fixed models produced unbiased and precise volume estimates, particularly when data were collected from a relatively homogeneous stand.

Forest is a large area of land spanning more than 0.5 hectares with trees that are at least 5 meters tall and have a canopy cover of more than 10 percent, or trees able to reach these thresholds.

The Food and Agriculture Organization of the United Nations provided this definition (Nations, 2019). In the context of uneven-aged stands, forests contain trees of different ages, sizes, and species, which complicates modeling because these stands do not follow simple, uniform growth patterns. Instead, they have heterogeneous spatial and temporal dynamics that can be modeled using statistical models (Acquah, 2023).

Fixed effect models are typically used in; even-aged (plantation) or managed stands with low variability, situations where all relevant variability can be explained by the fixed predictors. For example, in tropical forest inventory, fixed effect models have been used to estimate tree volume and height where data from a specific location or species were available and external variability was assumed negligible (Aigbe & Omokhua, 2015). Recent studies have validated the utility of fixed-effect models in forestry (Siipilehto, 2025). Zhang *et al.* (2024) showed that fixed-effect height-diameter models performed effectively in Northeastern China's Da Xing'an Mountains when stand-level parameters were controlled, reinforcing the model's strength in structurally consistent settings. These models also serve well in forest inventory contexts, particularly where data collection is limited to a single species or ecological zone, and where random sources of variability such as species or site are either known or negligible, one of the key advantages of fixed-effect models lies in their low computational demand, which proves beneficial when dealing with large-scale forest data (Xu, Q., Yang, F., Hu, S., He, X., & Hong, Y. 2024). However, recent evaluations caution against overlooking their limitations, Fixed-effect models can produce biased estimates when applied to diverse or uneven-aged forests, as they do not account for hierarchical variability that is often present in natural forest ecosystems (Xu, Q., Yang, F., Hu, S., He, X., & Hong, Y. 2024). Hence, the main aim of this

study was to estimate and compare well established fixed effect models for the effective management of the study site.

MATERIALS AND METHODS

Study area

The study was conducted at the Department of Forest Resources and Wildlife Management' Arboretum and Nursery in the Faculty of Agriculture University of Benin, located within the main campus in Ugbowo, Benin City, Edo State, Nigeria. The arboretum is established primarily for conservation, research, and educational purposes. It hosts a wide varieties of indigenous and exotic tree species representative of both tropical rainforest and savanna ecosystems (Osemeobo & Okunomo, 2017).

The coordinates of the arboretum are; Latitude 6°24'3.24" N longitude 5°37'25.68" E. Vegetation within the arboretum reflects a mixture of secondary forest species, pioneer species, and planted exotic trees. Some of the notable species commonly found in the arboretum include *Terminalia superba*, *Gmelina arborea*, *Tectona grandis*, and *Milicia excelsa* (Osemeobo & Okunomo, 2017). Human activities such as light foot traffic, periodic maintenance, and occasional research interventions have had minimal disturbance impact. As such, the University of Benin Arboretum provides a suitable and controlled environment for comparing fixed-effect modeling approaches in an uneven-aged stand. Geographically, the University of Benin is located in the Ovia North-East local government area of Edo state, within the humid tropical climatic zone of southern Nigeria. The climate is characterized by two distinct seasons: a wet season from April to October and a dry season from November to March, with an annual rainfall ranging between 1,500 mm and 2,000 mm (Awode, 2025). The area of the Department of Forestry and Wildlife measures

1140.1527 square meters, approximately 0.151 hectares.

Data Processing and Analysis

The process of transforming raw field data into useful information for comparing fixed effect models in predicting forest growth is known as data processing. The processed data served as the basis for comparing fixed effect models in predicting forest growth and stand dynamics in a mixed forest stand. The accuracy and reliability of the model outcomes will largely depend on the quality and efficiency of the data processing techniques employed during the project. Thus, excel was used to structure out the dataset templates of the trees from the study site. Model fitting and comparison was performed using “R” software which provides robust tools for implementing statistical models (R Core Team, 2022).

Collection of Data

The process involved tagging or placing a mark on the trees using marker, this was closely followed by measuring tree diameter, height of the trees and recording tree variables. Tree diameter and height are routinely measured in forest inventory because they are the fundamental variables from which other stand characteristics are derived (Ogana 2018). Diameter and height are key components for analysis of forest stand structure, estimation of volume (Gomez-Garcia *et al.* 2014), biomass, basal area, density and are commonly used as predictors of tree growth (West 2015). The data collected from the measurement process was recorded in the field notebook respectively. Total enumeration is feasible and preferred due to the small size of the study area, typically trees of DBH above 10cm were measured for this study. The following instruments were used; Diameter tape (DBH tape): Measures tree DBH at 1.3m height, haga altimeter: used to measure height, measuring tape, DBH stick (1.3m),

Markers: to mark/tag trees during the enumeration process, Field notebook: for recording observations and measurements, and a GPS device.

Data Cleaning

The dataset was cleaned before analysis, to ensure data quality for accurate model comparison. Records with missing or incomplete key measurements was identified. Cases with critical missing values was excluded, while minor gaps was addressed through imputation methods. The data was screened for duplicates, irregular intervals, and inconsistent units. Any records failing these checks was corrected or removed. Non-positive DBH or growth increments exceeding realistic thresholds, was excluded to prevent bias. Outliers was detected through standardized residuals from an initial fixed effect model; observations beyond ± 3 standard deviations from the mean residual was typically considered outliers (Sharma *et al.*, 2019; Zhang *et al.*, 2021). This cleaning process helped produced a reliable dataset for effective comparison of fixed effect models.

Data Collation

The data used for comparing models were collected by measuring the trees in the study area using a measuring tape, DBH stick, and a haga altimeter. DBH and total height are the essential measurements for predicting tree growth. The models include; Logistic, Chapman-Richard, Mitscherlich, Gompertz, polynomial and Hossfeld IV. This stage lays the ground work for strong statistical modelling and significant insights into predicting height using fixed effect models.

Table 1: Fixed Effect Models used for Prediction of Tree Growth in an uneven aged stand situated at the Faculty of Agriculture

Model	Model Equation	References
Chapman-Richard	$H_t = a(1 - e^{-Db})^c + E$	Chapman (1961) Richard (1959) Silva <i>et al</i> 2022
Mitscherlich	$H_t = a(1 - e^{-Db}) + E$	Mitscherlich, E.A. (1909)
Polynomial	$H_t = a + Db + cd^2 + Dd^3 + E$	Ratowsky, D.A. (1990)
Logistic model	$H_t = \frac{a}{1 + e^{-b(D - c)}}$	Parresol, B.R (1992)
Hossfeld IV	$H_t = \frac{a - b}{1 + e^{-b(D - c)}}$	Zeide <i>et al</i> 1993
Gompertz	$H_t = a.e^{b.e^{-Cd}}$	Hossfeld, F.(1980)
		Benjamin Gompertz <i>et al</i> 1993

Source:(Wang *et al* 2021, Adesuyi *et al* 2020, Silva *et al* 2022)

Where:

Ht = Tree total height in meters

D = DBH, diameter at breast height (cm)

a, b c, d = model parameters to be estimated

e = Base of the natural logarithm which is (2.71828)

E = experimental error or error term

Model Evaluation

The fixed effect models were assessed with the view of recommending those with good fit for further uses and to assess the predictive performance of the model. In order to compare the models, the Akaike information criterion (AIC), Root Mean Square Error (RMSE), the Bayesian Information Criterion (BIC) and coefficient of determination (R^2).

Akaike information criterion (AIC): To compare models with various parameter values, this is one of the most reliable criteria, it balances model fit and complexity. The smaller

the AIC value, the better the model (Wagenmakers & Farrell, 2020). It is defined as: $AIC = 2 \ln(L) + 2p$ (K. Ahmadi, S.J Alavi, 2016) Where; $\ln(L)$ is log likelihood and p is no of parameters

Root Mean Square Error (RMSE): It measures the spread of data and is a good indicator of precision. It represents the average difference between the observed and predicted values by the model. The value must be small, lower RMSE indicates higher accuracy. It is defined as:

$$RMSE = \sqrt{\frac{\sum_{i=1}^n (y_i - \hat{y}_i)^2}{n}} \quad (K. Ahmadi, S.J. Alavi, 2016)$$

Bayesian Information Criterion (BIC): This is similar to AIC but with a stronger penalty for complexity. For a model to be considered valid its BIC must be relatively low. It is defined as:

$$\text{BIC} = -n \ln(\text{RMSE}) + \ln(-n) \quad (\text{K. Ahmadi, S.J. Alavi, 2016})$$

Coefficient of determination (R^2): This is to measure of the proportion of variation in the dependent variable that is explained by the behavior of the independent variables or the model to be accepted, the R^2 value must be high ($>50\%$). It indicates the proportion of the variance in the dependent variable (height) that is predictable from the independent variable (diameter). It is defined as:

$$R^2 = 1 - \left[\frac{\sum_{i=1}^n (e_i)^2}{\sum_{i=1}^n (y_i - \bar{y})^2} \right] \quad (\text{K. Ahmadi, S.J. Alavi, 2016})$$

Where;

n = number of observation

H_i = observed value

p = number of parameters

$\hat{H}$ = estimated value

$\bar{H}$ = mean of observed values

The models were applied to the data set based on their suitability, model structure and their potential to accurately capture the growth dynamics in the study. Each model was parameterized estimating optimal parameters (a , b , and c) and their respective standard errors (SE). Subsequently, the models were statistically analyzed to assess their performance and goodness of fit based on the four main statistical test criterion above. RMSE evaluates the accuracy of a model by quantifying the disparities between the projected values and the observed values i.e minimum the values, higher the performance (Hodson, 2022). Coefficient of determination (R^2) provides a comprehensive measure of model reliability.

Data Validation

Data validation is paramount in ensuring the

integrity and reliability of the collected data, these processes are employed to guarantee accuracy throughout the data processing phase. It is highly essential to validate the models selected through assessment before their suitability can be introduced to forest management. Validation was done by comparing the models output with values observed on the field. In fitting models in forestry, the data obtained from the field are usually divided into two data sets; the calibration data set and the validation data set. The first data set was the calibration data set (70%) which is used for fitting the models, and the second data set is the validation data (30%) held back for model validation (Aishan *et al.*, 2016). The calibration data set is used to construct the models while the validation data set is used to test them.

Map of Study Area



Plate 1: Satellite View of the Study Area

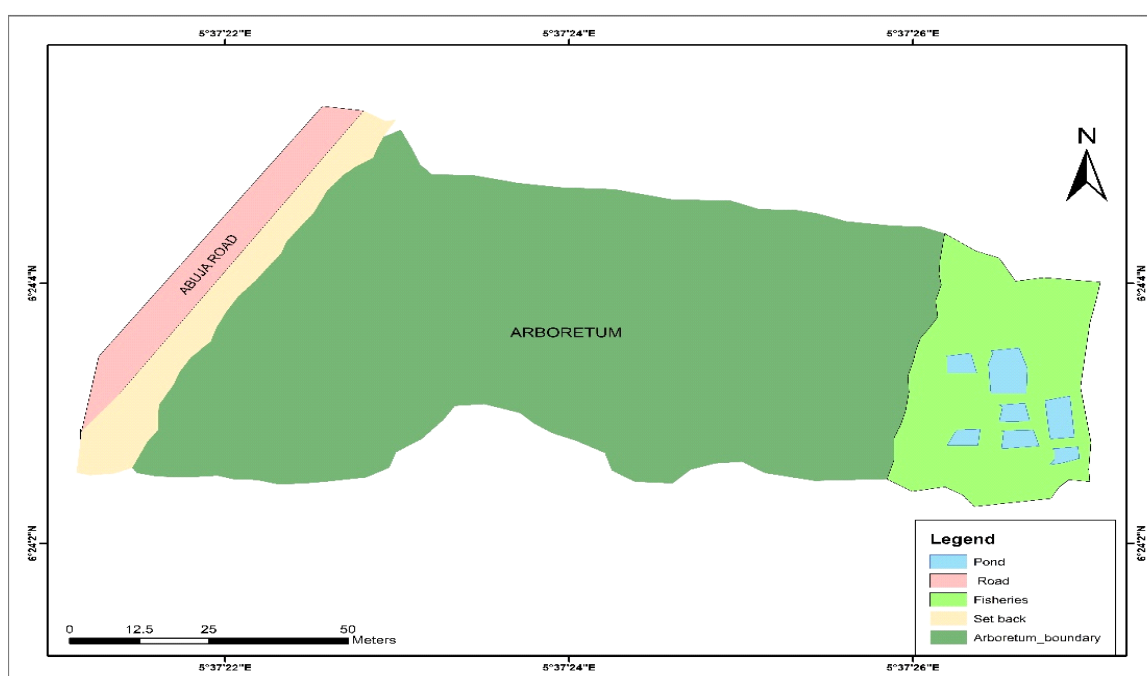


Figure 1: Map Representation of the Uneven Aged Stand

RESULTS AND DISCUSSION

Fixed effect model Comparison

The study site consisted of a 146 trees (≥ 10 cm dbh). Outlier(s) detection based on standardized residuals (± 3 SD) identified two trees ($\approx 1.4\%$ of the dataset), which were excluded prior to modelling, leaving 144 trees for calibration and validation. Chapman-Richard, Logistic,

Gompertz, Hossfeld IV, Polynomial fixed effect models, Akaike Information criterion (AIC), Bayesian Information Criterion (BIC) value, Coefficient of determination (R^2) and Root Mean Square Error (RMSE) value were used to evaluate the models and based on the evaluation the best model was selected.

Table 2: Summary statistics of the measured variables

	Mean	Standard deviation	Kurtosis	Minimum	Maximum
Db (cm)	32.14	11.25	-0.22	14.1	70.0
DBH (cm)	24.37	9.82	-0.12	10.0	52.0
Height (m)	18.64	6.11	-0.45	5.5	36.2

Source:(Data collection from field work)

Table 3: Correlation Table

	DB (cm)	DBH (cm)	Height(m)
DB (cm)	1.000		
DBH (cm)	0.925	1.000	
Height (m)	0.844	0.812	1.000

Source:(Data collection from Field work)

The results of the fitted fixed-effect models are summarized in table 4 below. Models resulting with the highest coefficient of determination (R^2), the least RMSE, and the smallest values of AIC and BIC respectively were selected as the best model. The six candidate fixed effect models (Mitscherlich, Chapman-Richards, Gompertz, Logistic (3p), Hossfeld IV, and polynomial model) were fitted using the calibration datasets (70% of observations). Model performance was evaluated on both

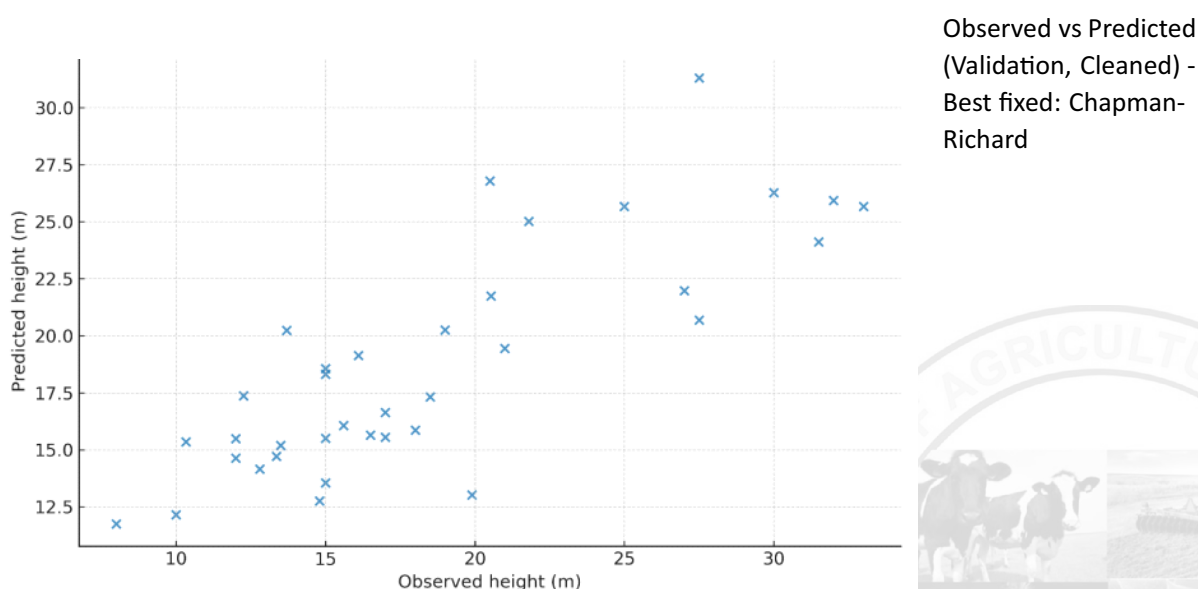
calibration and validation datasets using RMSE, R^2 , AIC, and BIC. The Chapman-Richard model provided the best validation performance among the fixed effect models, with highest R^2 ; 0.7619, least RMSE; 4.4978, smallest AIC and BIC; 330.7748 & 334.2812 respectively. On the other hand, Hossfeld IV gave the worst validation performance, with the highest RMSE; 22.561, least R^2 ; -4.9897, highest AIC and BIC values; 679.1043 & 687.1507 respectively.

Table 4: Showing the Calibration data sets

Models	Parameters β_0 , β_1 , β_2	RMSE	R^2	AIC	BIC
Logistic	38.44, 3.95, 0.05	4.5771	0.7535	334.549	342.5954
Gompertz	40.82, 1.86, 0.03	4.5404	0.7574	332.8097	340.8561
Mitscherlich	40.88, 0.02	4.5421	0.7572	330.8909	336.2551
Polynomial	2.43, 0.80, 0.07	4.502	0.7615	332.9789	343.7075
Chapman-Richards	52.81, 0.01, 0.75	4.4978	0.7619	330.7748	334.2812
Hossfeld IV	32.67, 10.12, 22.561 365.86		-4.9897	679.1043	687.1507

Table 5: Showing validation datasets for the fixed effect models

Model	RMSE	R^2	AIC	BIC
Logistic (3p)	3.9293	0.6444	103.1636	107.9141
Gompertz	3.8661	0.6558	103.3623	108.1129
Mitscherlich	3.9142	0.6472	104.5293	109.2799
Polynomial	3.9212	0.6459	106.3803	112.7144
Chpman-Richard	3.8555	0.6577	102.2513	105.4183
Hossfeld IV	19.6822	-7.9217	220.5395	225.290


Figure 2: scattered plot graph showing observed vs predicted values for a fixed-effect model

The scatter plot graph is a visual assessment of the predictive ability of the model, it shows how well the model's prediction match the actual tree data. Each point on a plot represents a single tree (Zhang, 2015; Mehtatalon & Lappi, 2020).

DISCUSSION

Tree height and diameter at breast height are critical parameters in modelling, to understand forest structure and productivity, developing and selecting effective models for tree height estimation is essential in forest management (Chenge, 2021). The fixed effect models were able to describe the relationship between height and diameter at breast height. Height and diameter has been used by many authors (Egonmwan, 2022; Ogana, 2019; Egonmwan & Orukpe, 2024) to give information about the stocking of the forest and for future prediction. Selection of the best models are based on appropriate criteria such as AIC, BIC, RMSE and R^2 . The summary of the growth variables was listed in table 1. The minimum and maximum diameter at breast height was 10.0cm and 52.0cm respectively, the minimum height was 5.5m while the maximum height was 36.2m. The standard deviation for diameter at breast height was 9.2m, the descriptive statistics showed moderate variation in trees dimensions, as reflected in the standard deviations. The correlation analysis demonstrated strong positive associations among DB, DBH, and height. The fixed-effect models used in this research were also used in the study carried out by Wan Razali WM *et al.* 2015 to predict early height growth of forest trees planted in Sarawak, Malaysia. The results of this research agrees with the work of Andrade *et al* 2018 and that of Silva *et al* 2022, in which the Chapman-Richard fixed effect model like in this study, was the best for describing height-diameter relationships in Tocantis Native forests Brazil. The fixed effect model in figure 2 is useful in estimating height

from diameter at base, diameter and breast height, but its major limitation is its inability to account for hierarchical structure (e.g species groups within a stand, trees nested within plots Patricio, 2022).

CONCLUSION

This study compared the performance of six fixed-effect growth H-D models; Logistic, Chapman-Richards, Mitscherlich, Gompertz, Polynomial, and Hossfeld IV in estimating tree growth within a mixed forest stand in Edo State, Nigeria. They provide estimates for dependent variables based on measured variables, while assuming other influences are constant (West *et al* 2025). Its inability to account for stand variation make it unrealistic in highly heterogeneous stand like the uneven aged stand used for this research (Saunders & Wagner 2008; Zhang *et al.*, 2021). This modelling approach is easy to implement in forest inventories, making it accessible to practitioners without advanced statistical tools (Zhang *et al* 2021). The results demonstrated clear variations in model accuracy, highlighting the importance of selecting an appropriate model for reliable forest growth prediction. Among the evaluated models, the Chapman-Richards model consistently provided the best fit, as evidenced by its higher coefficient of determination and lower RMSE, AIC, and BIC values. Its superior performance underscores its capacity to capture the nonlinear and asymptotic growth patterns typical of mixed tropical forests. The findings affirm that the Chapman-Richards function is a robust and reliable tool for modeling tree growth in heterogeneous forest conditions, offering improved precision for forest managers, researchers, and policymakers. By adopting this model, forest management planning such as yield estimation, harvesting schedules, and long-term sustainability assessments can be significantly

enhanced. Overall, this study contributes valuable empirical evidence to growth-model selection in Nigerian mixed forests and reinforces the role of well-fitted growth models in promoting sustainable forest management across diverse ecological landscapes.

RECOMMENDATIONS

The Chapman–Richards fixed-effect model is strongly recommended for height–diameter (H–D) growth prediction in mixed forest stands due to its superior performance relative to the other evaluated models. Its ability to effectively represent nonlinear and asymptotic growth trends makes it especially suitable for tropical forests characterized by diverse species composition and complex growth dynamics. Forest managers, inventory planners, and researchers should prioritize this model when reliable height estimation is required but direct measurement is impractical or costly. Despite the practical advantages of fixed-effect models, such as ease of implementation and minimal data requirements, their limitation in accounting for stand-level variability should be carefully considered. In highly heterogeneous and uneven-aged stands, as used in this study, fixed-effect models may oversimplify growth processes. Therefore, it is recommended that results derived from these models be applied with caution, particularly when used for long-term projections or management decisions involving structurally complex forests. Future research should explore the integration of mixed-effects or hierarchical modelling approaches that incorporate random effects to better capture stand-level and species-specific variability. Such models would likely improve

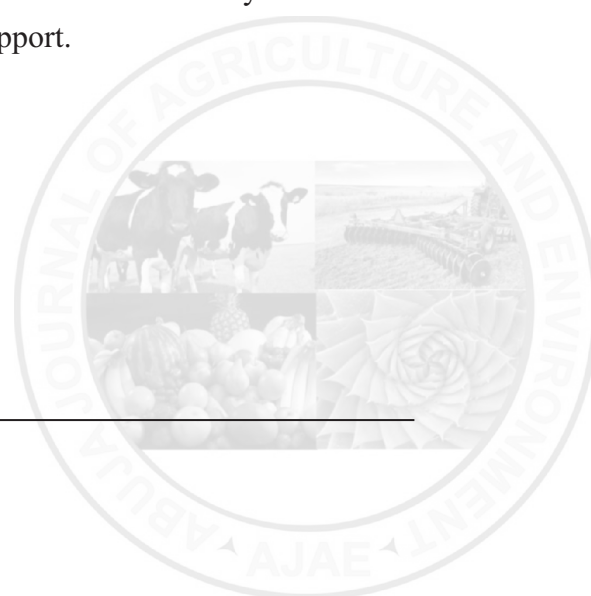
prediction accuracy and realism in heterogeneous forest conditions while building upon the baseline performance demonstrated by fixed-effect models. Additionally, expanding datasets to include longer-term monitoring, wider diameter ranges, and more ecological covariates such as site quality, competition indices, and soil characteristics is recommended. This would enhance model robustness and allow for broader applicability across different forest types and management regimes. Finally, capacity building for forest practitioners is encouraged to promote informed model selection and application. While fixed-effect models remain valuable tools due to their accessibility, improved understanding of their assumptions and limitations will support better decision-making and sustainable forest management outcomes. Overall, the study underscores the importance of model evaluation and selection in forest growth prediction and provides a strong empirical basis for the adoption of the Chapman–Richards model in mixed tropical forest stands, while also highlighting pathways for methodological advancement in future studies.

Competing Interest

The authors have declared that no competing interests exist.

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