EVALUATION OF TEN (3-PARAMETERS) NON LINEAR MODELS FOR PREDICTION OF *Gmelina arborea Roxb.* STAND IN SOUTHERN WEST, NIGERIA

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ABSTRACT

Accurate modeling of the height-diameter (H-D) relation is significant for effective forest management, especially in tropical plantations where data on height are normally limited. In this study, the relative efficiency of ten three-parameter non-linear models to predict tree height from diameter at breast height (DBH) was evaluated for Gmelina arborea plantation in Southwestern Nigeria. The models were also compared and contrasted on the basis of various statistical measures, such as; Root Mean Square Error (RMSE), Coefficient of Determination (R^2), Mean Absolute Error (MAE), Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC), residuals plot and Dense Ranking to determine their goodness-of-fit and predictive accuracy. Results were manifested with slight variation in performance of the models, of which the Korf H-D model was most suited for height prediction due to its overall better accuracy across all measures of evaluation. It had the smallest RMSE, BIC, AIC, MAE and the highest R^2 of 3.918, 1630.585, 1617.445, 3.046 and 0.816 (81.6%) respectively. It had high compatibility of model complexity with predictive ability, and thus it was a potential candidate for use in real application in forest inventory and growth prediction. Although the other models also gave a good explanation of the dataset, the Schnute H-D model was the poorest, manifested with the poorest fit and highest error rates among all the models tried. The study highlights the significance of correct model choice in forest modeling and recommends the employment of the Korf H-D model in estimating tree height in Gmelina arborea plantation in similar ecological regions. Future research can explore the integration of additional site variables or the application of mixed-effects and machine learning models to enhance predictability.

Keywords: Korf H-D model, 3-parameters, lignin, DBH, Juvenile wood and pulp

INTRODUCTION

The physicochemical properties of wood, for example its lignin and cellulose content, density, are determined by the diameter and height of the trunks (Ramírez, 2025). Tree diameter is a significant tree attribute for sustainable forest management and an important characteristic in resource inventory. Diameter measurement can be made along the tree trunk, branch, or log which could either be determined as over-bark or under-bark diameter. Diameter at breast height (Dbh) is the most common diameter measurement and is taken at 1.3 m above ground level on a standing tree (Matthew, 2024). Measurement of tree diameter at breast height is relatively simple, accurate and with low cost (Ferraz- Filho *et al.,* 2018; Corral-Rivas *et al.,* 2019).

Tree height (ht) is an important element in forest management and is used to characterize forest structure, estimate volume and biomass, and assess site quality (Picard, 2025; Chen, 2024). Unlike diameter determination, tree height measurement is difficult, time consuming and expensive (Mehtätalo *et al.,* 2015; Ozcelik *et al.,* 2018). Owing to the associated problem with tree height measurement, only sub-sample of trees is measured. Tree height is prone to measurement errors as it requires observing the top of the tree to make precise estimates (Maltamo, 2023), which is often complicated in closed-canopy conditions, with broken trees, and in complex topographic positions (**Gatziolis**, 2010). Owing to the associated problem with tree height measurement, only sub-sample of trees is measured. Thus, height-diameter (H-D) models are often used to estimate the height of trees for which the diameters have been measured instead of direct measurements (Kalbi *et al.,* 2017; Mehtätalo *et al.,* 2015).

Most H-D functions have been developed for forest plantations (Huang, 2025; Mehtätalo, 2015). Diameter at breast height and height are considered the main variables for determining trunk volumes and biomass. However, the relationship between the diameter and height of a tree varies between stands (Chenge, 2023) because it depends on stand characteristics such as density and site index (Duan and Zhang 2025). Moreover, the h-d relationship also varies over time within the same stand (Kılıç, 2024). Such considerations indicate that stand variables should be used to construct generalized functions that represent all possible conditions in forest stands (Kılıç, 2024). Most studies have relied on the use of height-diameter (h-d) models to estimate the height of trees. Some common h-d models that have been used in quantitative forestry include: Logistic (Egonmwan, 2022), Korf (Lundqvist 1957), Chapman-Richards (Richards 1959), Curtis (Kılıç, 2024), Wykoff (Wykoff *et al*. 1982) etc. Many published forestry literatures exist on h-d models (Egonmwan, 2022, Mehtätalo *et al*. 2015, Eby *et al*. 2017, Ogana 2018) for plantation and natural forest stand.

Gmelina arborea (Roxb.) of Verbenaceae family is a broad to large deciduous tree with an unarmed, straight and unbranched trunk. The species possesses a spreading, broad, deeply shaded canopy that has numerous branches. The mature trees are capable of attaining a height of over 30 meters with a maximum diameter of the trunk being 4.5 meters. The bark is usually smooth, exhibiting pale ashy-grey to yellowish-grey coloration, often having black patches and sharply defined corky circular lenticels (Kılıç, 2024). The inflorescence of *G. arborea* is profuse and scented, flowers being reddish, brownish, or yellowish in color. These are found in terminal and axillary cymes having one to three flowers in a cyme, borne on the branch lets of an 8–40 cm long panicle (Liu, 2019). The seeds are typically 1, but exceptionally 3, lenticular in shape, and are without endosperm (exalbuminous). Genus Gmelina is dedicated to Johann Christoph Gmelin, a renowned 18th-century German botanist. Species epithet arborea comes from the Latin arbor meaning "tree," referring to tree-like habit of the plant (Rojas, 2025). Accurate modeling of the height-diameter (H-D) relationship is essential for effective forest management, especially in tropical plantations where height measurements are often limited. This study aimed to evaluate and identify the most suitable model for accurate and efficient stand-level height prediction from tree diameter for the efficient and sustainable management of the *Gmelina arborea* plantation in Oluwa forest reserve.

METHODOLOGY

Study Area

The data used in this study were from the *Gmelina arborea* plantation (about 9 ha in size) in the Oluwa Forest Reserve (FR) of Southwestern Nigeria. The Oluwa FR is situated between 6º 55′ and 7º 20′ N and longitude 3º 45′ and 4º 32′ E, and occupies an area of 87,816 ha. “Oluwa FR has an annual rainfall in the range 1700 to 2200 mm, an average annual temperature of 26°C, and a mean elevation of 123 m above sea level”. “Establishment of large-scale plantations in the reserve started in early 1960s. *Gmelina arborea* Roxb. and *Tectona grandis* L.f. are the dominant plantation species in Oluwa FR”.

Data Collection

Data were collected from twenty-five temporary sample plots of 20m × 20m in five stands of the *Gmelina* arborea plantation. “Diameter measurements of all trees within each plot (outside bark) at breast height 1.3 m above ground were measured. Dead and badly defective trees were not included in the data collection process. Their corresponding height (H) measurements were also taken with Spiegel relaskop. The measured variables were used to calculate the stand basal area (m^2) and the observed Newton's tree volume (m^3). Selective sampling design method was adopted and a total of five hundred and ninety (590) trees with desirable characteristics were purposively selected for enumeration. Diameter at different points along the stem such as at the base, breast height; mid-point and top diameters were taken from the forest reserve.

3-Parameters Non Linear Models

Ten well established three parameters H-D models were used in this study to check for their suitability in fitting the data sets of the *Gmelina arborea* stand in Oluwa forest reserve. All the models were parameterized so that parameter *bo* defines the scale of the H–D models, which was the major dimension of variability among sample plots (Egonmwan 2022). Models that expressed tree height without transformations were used in other to avoid problems with back transformation bias. The datasets for this study were handled with R-script using package “nlme” and “minpack.lm” for robust analysis of *R*-environment (R Core Team 2017), to generate parameter estimates, model evaluation and visualization.

Fitting models

Table 1: Non Linear H-D models

Model Name	Model Form	References	Eq.
Schnute	$= 1.3^{1} + \quad 2^{1} - 1.3^{1} \frac{1- \quad - \quad 0}{1- \quad - \quad 2- \quad 0}^{\frac{1}{1}}$	Schnute 1981	[1]
Logistic	$= 1.3 + \frac{}{1 + {}_{1}{}^{-1}. \quad {}^{-2}}$	Huang *et al.*, 1992	[2]
Weibull	$= 1.3 + \quad 1 - \quad {}^{-1.} \quad {}^{2}$	Yang *et al*. 1978	[3]
Chapman/Richards	$= 1.3 + \quad 1 - \quad {}^{-1.} \quad {}^{2}$	Chapman1961, Richards 1959	[4]
Korf	$= 1.3 + \quad . \quad {}^{-1.} \quad {}^{-2}$	Stage 1963	[5]
Peschel	$= 1.3 + \frac{}{1 + \frac{1}{1 \quad 2}}$	Peschel (1938)	[6]
Hossfeld	$= 1.3 + \frac{}{{}_{1} + {}_{2}}$	Hossfeld (1822)	[7]
Strand	$= 1.3 + \frac{2}{2 + {}_{1} + {}_{2}}$	Strand (1964)	[8]
Sibbesen	$= 1.3 + \quad {}_{1} \quad {}^{-2}$	Huang *et al.* (1992)	[9]
Power	$= \quad + {}_{1} \times (^{2}$	Fang & Bailey (1998)	[10]

b_o, b_1, b_2 are parameter estimates from the H-D models, H is total height, D is Diameter at breast height and *e* is exponential

Evaluation statistics

The models were evaluated in order to test their plausibility and also recommend them for onward use. The models were assessed based on; Root Mean Square Error (RMSE), R-squared (R^2), Bayesian Information Criterion (BIC), Akaike Information Criterion (AIC), and Mean Absolute Error (MAE). Dense ranking system was used to give a final judgement on the best model. Model selection was based on the criterion that the higher the R^2, smaller the values of (RMSE), (AIC), (BIC), and (MAE) the better the model. Residuals were also graphically examined to check for any trend. The smaller the rank the better the performance of the model. The evaluation statistics are represented below as:

Table 2: Equation forms of evaluation statistics

Name	Form	Eq
Root Mean Square Error	$= \sqrt{\frac{\sum_{=1} (-)^2}{}}$	[11]
Coefficient of Determination	$R^2 = 1 - \frac{}{}$	[12]
Bayesian Information Criterion	$= \ln \frac{RSS}{n} + \ln$	[13]
Akaike Information Criterion	$= \ln \frac{}{} + 2$	[14]
Mean Absolute Error	$= \frac{1}{} \sum_{=1} -$	[15]

Where: R^2= Coefficient of determination, SS_{res} = Residual sum of squares, SS_{tot} = Total sum of squares, RSS = residual sum of square, n = sample size, p = number of parameters; Yi is the observed value, is the theoretical value predicted by the model and − = absolute error of the ^{h} data point.

RESULTS

The summary statistics of the data set showed that the study site is rich in nutrient as the minimum and maximum height are 5.70m and 49.70m respectively, while the (min and max) DBH were 10.10 and 52.30cm (Table 3), respectively. Mean values, standard deviation, skewness and kurtosis of the entire calibrating data set are indicated in the table below:

Table 3: Summary of all the modelling datasets

Statistics	H (m)	DBH (cm)	CL (m)	BA (m^2)	VOL (m^3)
Minimum	5.70	10.10	0.80	0.008	0.035
Maximum	49.70	52.30	6.60	0.215	4.685
Mean	22.08	24.55	3.10	0.054	0.674
Standard Deviation	9.15	9.05	1.19	0.039	0.671
Skewness	0.14	0.46	0.57	1.125	1.933
Kurtosis	-0.75	-0.56	-0.24	1.038	5.050
N = 590					

The tables 4 and 5 below provide the values of the ten H-D models, parameter estimates along the various evaluation test results (Table 4) and the dense ranking ranking values (Table 5).

Table 4: Model parameter estimates and test statistics result

	Parameter Estimates			Evaluation Statistics				
Model Name	b_o	b_1	b_2	**RMSE**	R^2	**BIC**	**AIC**	**MAE**
Schnute	-0.0241	2.1904	44.3505	3.946	0.814	3319.656	3302.136	3.113
Logistic	40.3743	0.1057	22.1559	3.983	0.810	1649.875	1636.735	3.067
Weibull	0.045	0.0418	0.1582	3.938	0.815	1636.492	1623.352	3.055
Chapman/Richards	48.5181	0.0453	2.0261	3.930	0.815	1634.093	1620.953	3.050
Korf	98.6707	14.4713	0.7052	3.918	0.816	1630.585	1617.445	3.046
Peschel	56.4151	0.0020	1.7883	3.928	0.815	1633.636	1620.496	3.049
Hossfeld	1.7883	8.7218	0.0177	3.928	0.815	1633.636	1620.496	3.049
Strand	0.0151	0.2900	11.6693	3.926	0.816	1632.805	1619.665	3.051
Sibbesen	0.0100	4.1634	0.1726	3.926	0.815	1632.934	1619.794	3.059
Power	-32.4363	15.2211	0.4038	3.926	0.815	1632.934	1619.794	3.054

Table 5: Dense Ranking Results

Model	**RMSE**	**Dense Rank**	R^2	**Dense Rank**	**BIC**	**Dense Rank**	**AIC**	**Dense Rank**	**MAE**	**Dense Rank**
Schnute	3.946	6	0.814	2	3319.656	8	3302.136	8	3.113	9
Logistic	3.983	7	0.810	1	1649.875	7	1636.735	7	3.067	8
Weibull	3.938	5	0.815	3	1636.492	6	1623.352	6	3.055	6
Chapman/Richards	3.930	4	0.815	3	1634.093	5	1620.953	5	3.050	3
*Korf	3.918	1*	0.816	4*	1630.585	1*	1617.445	1*	3.046	1*
Peschel	3.928	3	0.815	3	1633.636	4	1620.496	4	3.049	2
Hossfeld	3.928	3	0.815	3	1633.636	4	1620.496	4	3.049	2
Strand	3.926	2	0.816	4	1632.805	2	1619.665	2	3.051	4
Sibbesen	3.926	2	0.815	3	1632.934	3	1619.794	3	3.059	7
Power	3.926	2	0.815	3	1632.934	3	1619.794	3	3.054	5

*Korf H-D is selected based on model evaluation criterion and Dense Ranking

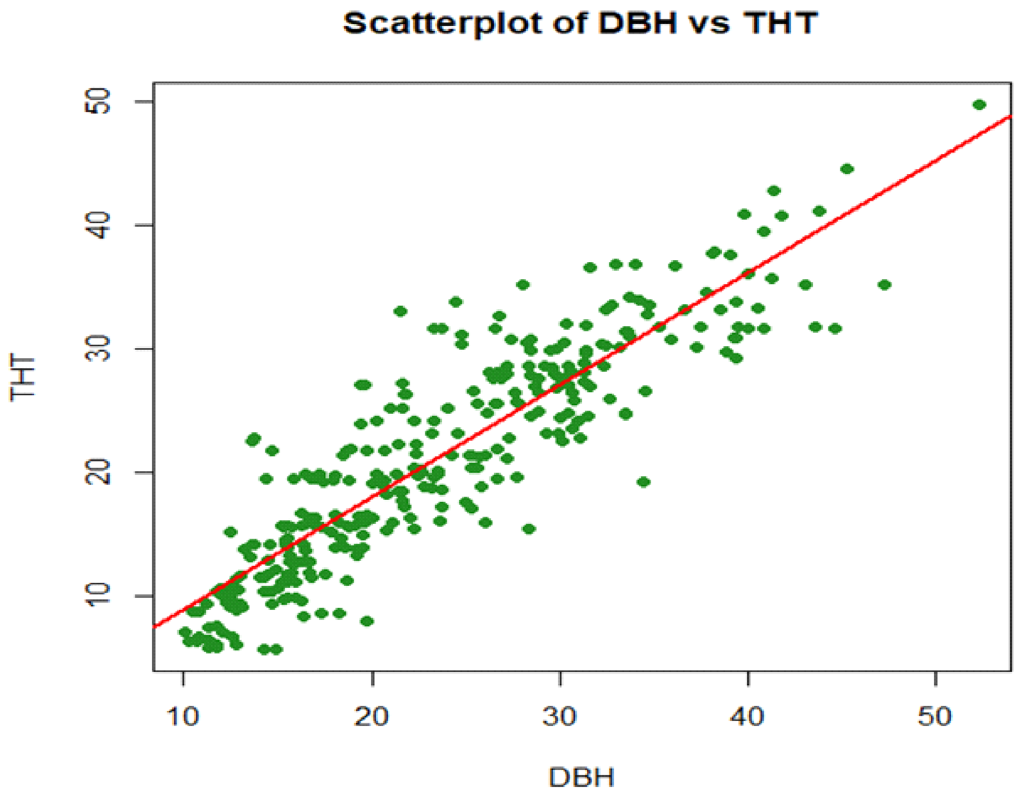

Figure 1: Scatter plot of the data set showing spread

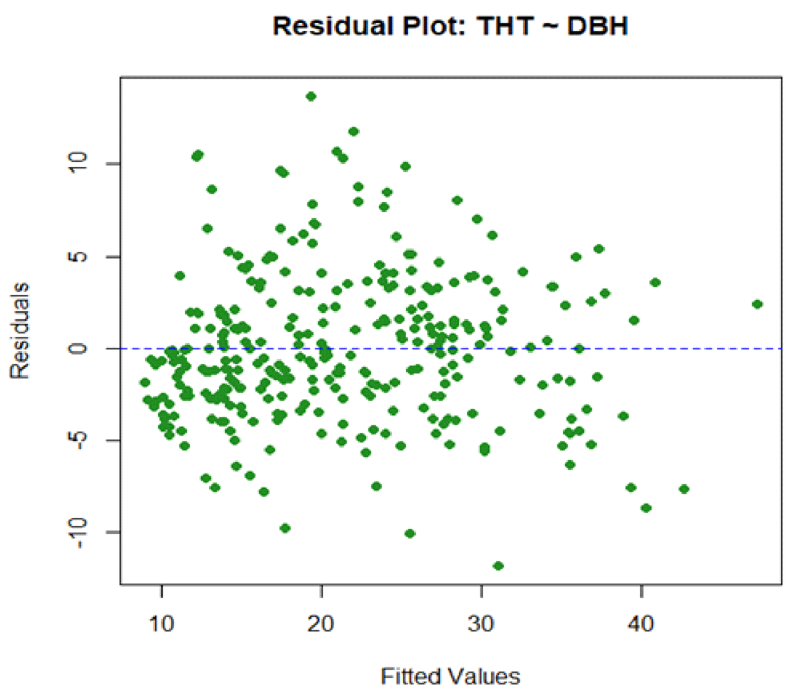

Figure 2: Residual plot of the data set with fitted values

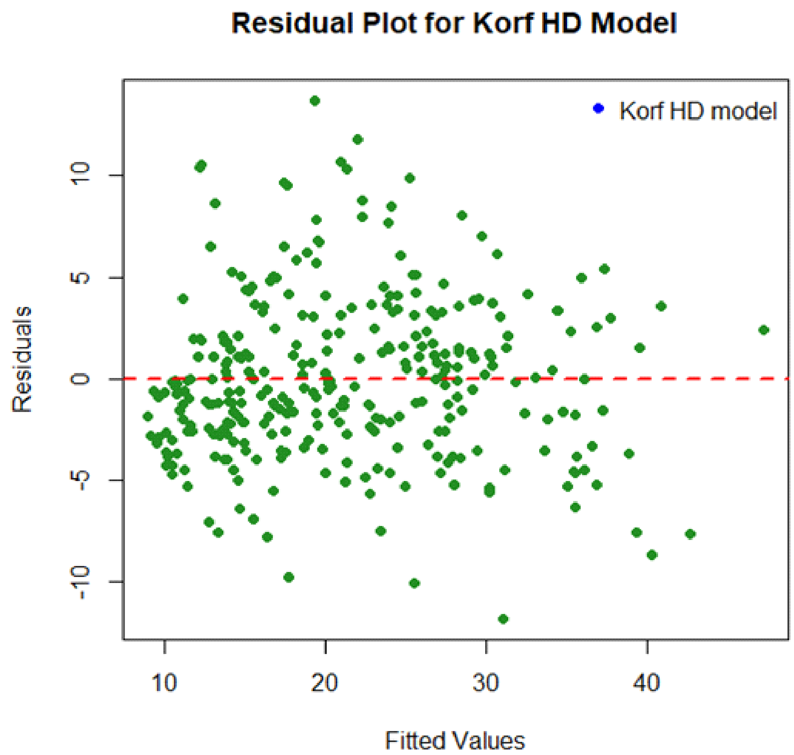

Figure 3: Residual plot of the selected Korf H-D mode

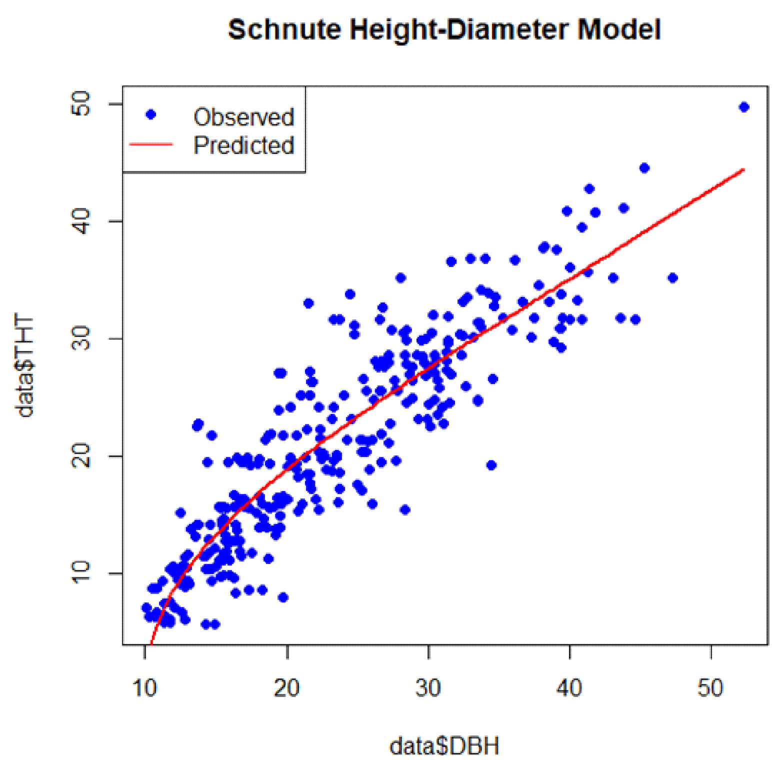

Figure 4: Scatter plot of the Schnute H-D model

DISCUSSION

The ten three parameters H-D models for predicting height have been developed and evaluated based on evaluation test criterion such as; RMSE, R^2, BIC, AIC and MAE, the result is as presented in Table 4 and a dense ranking system (Table 5) was also used to put a finality in adjudging the candidate (selected) H-D model. The result indicated that the 3-parameter Korf H-D model provided the best prediction for tree height in the *Gmelina arborea* plantation of the Oluwa forest reserve with RMSE, R^2, BIC, AIC and MAE evaluation criterion values of 3.918, 0.816 (81.6%), 1630.585, 1617.445 and 3.046 respectively hence has been selected as the candidate H-D model for this site. Closely following the Korf model were the Strand, Sibbesen and Power H-D models in terms of a smaller RMSE value as they all had equal RMSE values which was used as the highest penalty test criterion for this study. In fact, only marginal differences exist in the fit indices of the H-D models.

In turn, the Peschel and Hossfeld H-D models had the same results across all five evaluation test criterion for both models. As a rule of thumb, two models are said to be the same if the difference in their AICs values is less than two (Tewari & Singh 2018). The difference in AIC values for five of the H-D models (Peschel, Hossfeld, Strand, Sibbesen and Power) were less than two. All models in this study had p-value < 0.05. Schunute H-D model was the worse model in predicting height for the *Gmelina arborea* stand in Oluwa forest reserve, with a high RMSE, R^2, BIC, AIC and MAE of 3.946, 0.814, 3319.656, 3302.136, and 3.113 respectively. A Pearson correlation of 0.8978 (which was close to +1) between DBH and total height indicate a strong, positive linear relationship, which suggests that trees with larger diameters tend to be significantly taller (which is a common and biologically expected pattern), and the relationship is statistically meaningful. This strong correlation suggests that **DBH could be a good predictor of tree height (THT).** The scatter plot revealed that the data set appears to be **fairly homoscedastic and residual plot revealed that no statistical assumptions such homoscedasticity, independence and normality were violated.**

It has been stated that for any appropriate H-D model, the asymptotic t-statistic for each coefficient has to be significant, the model RMSE has to be small and the standardized residual plot should show approximately homogeneous variance over the full range of predicted values (Priyanka, 2022).

The result from this study was different from the study of (Ogana, 2020), who adjudged fitted Näslund height-diameter model as the best candidate for the *Gmelina arborea* plantation in Oluwa forest Reserve, while others were model specific in their study using Weibull model (Ekpa *et al.,* 2014), as a single model, giving rise to bias, as this did not allow for model consistency, flexibility and comparison. In the study of (Egonmwan & Ogana 2020) in Oluwa Forest Reserve, though in the Teak plantation, the Logistic H-D model, gave the best fit for the stand, while in the work of (Lebedev, 2020), Richard Chapman 3-parameter gave the best fit. The conclusion in the findings of (Egonmwan, 2022), the Logistic H-D model with 3-parameters was confirmed to provide a secure estimate of total tree height for *Tectona grandis* stand in Oluwa Forest Reserve. In the study of (Ismail, 2025) in which model performance was assessed using root mean square error (RMSE), mean absolute error (MAE), and the coefficient of determination (R^2), similar to this study in model assessment, results identified the Ratkowsky model as the best performer.

Sharma, 2009 opined that Hossfeld's model accounted for the largest proportion of height variations (R^2_{adj} = 86%), and appeared to be biologically most realistic. A drawback of this conclusion was that a single evaluation statistical criterion was used to reach this conclusion which is not statistically and scientifically balance. A high R^2 does not necessarily account for a better fit or suitability in model evaluation but it can be **misleading**, as a high R^2 does not guarantee good predictive accuracy if residuals are biased. R^2 cannot be used to select the best model because it increases as predictors are added i.e. overfitting is guaranteed. Hence, R^2 cannot be used as a sole indicator for determining the best model (Egonmwan & Whitney, 2024). Multiple test evaluation criterion should be employed. The study of (Raptis, 2024), the 3-parameter Hossfeld function explained approximately 84.0% of the total height variance in the case of King Boris fir and Scots pine species. There is no basis for comparing the fitted values of the study at hand and others reported in forestry literatures. As the parameters of the models are locations specific and more importantly heavily depends on the quality of data used in fitting the H-D models (Egonmwan, 2022).

The analysis of the plot of residual and predicted values demonstrated that there was little or no systematic bias towards over-or underestimation of the tree total height (Egonmwan, 2022), (Fig. 2 & 3). The assumption of ordinary nonlinear least squares regression was not violated in the models as the residual analysis showed almost homogenous variance (homoscedasticity) over the range of the predicted values and there were no systematic patterns. The residual plot of the height and diameter at breast height and the Korf H-D model (the best candidate) were only presented. The Korf model have met all five evaluation criterion and thus is recommended as the best 3-parameter H-D model that best fit the data set of the *Gmelina arborea* plantation in Oluwa forest Reserve in comparison with the other H-D models which also provided a better explanation in predicting height with the exception of the Schnute H-D which was the worse model based on the test criterion.

CONCLUSION AND RECOMMENDATION

This study evaluated ten three-parameter non-linear models for predicting the height-diameter (H-D) relationship of *Gmelina arborea* Roxb. stand in Southwestern Nigeria, with the primary objective of identifying the most suitable model for accurate and efficient stand-level height prediction. The non-linear modeling approach remains vital in forest biometrics due to its capacity to represent complex biological growth processes more effectively than linear methods. The analysis revealed varying performance across the models, with the Korf H-D model emerging as the most robust and reliable based on goodness-of-fit statistics, including root mean square error (RMSE), coefficient of determination (R^2), Bayesian Information Criterion (BIC), Akaike Information Criterion (AIC), Mean Absolute Error (MAE) and residual analysis. The model demonstrated superior flexibility and predictive accuracy across the diameter classes and site conditions sampled, thereby making it a preferred tool for forest managers and researchers in the region. Conversely, the Schnute H-D model displayed the least performance among the evaluated models. Its poor fit, higher prediction errors, and inconsistent residual patterns suggest limited suitability for modeling the H-D relationship of *Gmelina arborea* within the studied ecological context.

Based on these findings, the Korf H-D model is strongly recommended for practical applications in forest inventory, growth projections, and silvicultural planning involving *Gmelina arborea* in Southwestern Nigeria. Its application can enhance the precision of height estimations where diameter at breast height (DBH) is known, thereby reducing the need for destructive sampling and improving operational efficiency.

Future studies may consider expanding this evaluation framework by incorporating site-specific variables, mixed-effects modeling, or machine learning approaches to improve model generalizability across broader ecological zones. Continuous model validation using independent datasets is also encouraged to ensure long-term reliability and adaptability to changing forest conditions

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